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Relativistic Quarks Bound with a One-Body Tensor Potential

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A dipole fit to electromagnetic form factors is used to determine a quark density ρ in the nucleon. A radial tensor potential is used to bind the quarks into states of good J, J_Z, and parity. The tensor potential radial component is taken to satisfy the equation $T' = T_0\rho$, where T_0 is a parameter of the model. This linear divergence equation can be simultaneously solved with the Dirac equation for the bound quark wave functions. A self-consistent solution is possible where the mass density used as the source for the binding potential is the same as that determined from the solution for the quark wave functions.

1. INTRODUCTION

In the QCD theory, the quarks interact with the exchange of gluons characterized by a color vector potential. The quarks serve as the source terms of the color vector potential. The quarks in any system are assumed to form an $SU(3)$ color singlet. A gluon–gluon interaction makes the color vector potential equations nonlinear. The proton is modeled here as three interacting colored quarks. The color charge density from the quarks is taken as proportional to the electric charge density inferred from electromagnetic form factor measurements. Electromagnetic form factors of the nucleons are well reproduced by the dipole fit. From this one can infer an exponentially damped radial distribution of electric charge, and also for the quark color charge density serving as the source for the QCD color vector potentials. A tensor potential is used here to create bound quark states without tunneling possible via a Klein paradox.

The quarks are described here by a solution of the Dirac equation in a one-body external potential. This one-body potential is a shell model potential

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resulting from the quark–quark interactions presumably described by QCD. It is not certain if the one-body potentials used here should be colorless or not. The potentials are assumed colored, and coupled to the quark color charge density. Miller (1975) has shown that the only one-body external potential components capable of satisfying simultaneously the requirements of angular momentum conservation, Hermiticity, and time reversal invariance are a scalar, a time component of a vector, or a radial component of a tensor potential. The quark binding potential is assumed to be the radial component of a tensor potential in this paper. The other possibilities are neglected. The QCD equations utilize a color vector potential. The solution of the QCD equations is not at hand, but is assumed to not result in a scalar external potential. A possibly more likely result of solving the QCD equations would be the time component of a vector potential. The time component of a one-body vector potential, however, suffers from the Klein paradox, and by itself, cannot confine quarks (Su and Yuhong, 1984; Su and Ma, 1986; Galic, 1988).

The radial component of a tensor potential can confine quarks as a relativistic interaction without the Klein paradox (Dominquez-Adame, 1992). Bachas (1986) has shown that a heavy quark–antiquark potential must be everywhere attractive and a nondecreasing function of their separation. This has experimentally observable consequences in the level ordering of potential models (Baumgartner *et al.*, 1985), using the nonrelativistic Schrödinger equation. It is speculated here that the one-body quark potential should also be an attractive nondecreasing function of their separation. This is consistent with the Lipkin rule, based on a one-gluon exchange analysis, which states that the quark–quark potential is one-half of the quark–antiquark potential. Self-consistency requires that the solution of the Dirac equation for the quark wave functions match the color charge density used as the source in determining the potential that binds the quarks.

2. THEORY

The most general Dirac equation consistent with good total angular momentum and z component, parity, Hermiticity, and time-reversal invariance is

$$\{c\boldsymbol{\alpha}\cdot\mathbf{p} + \beta[mc^2 + U_s(r) + \gamma^0 U_v(r) - \gamma^0\boldsymbol{\gamma}\cdot\mathbf{U}_t(r)]\}\Psi = E\Psi \tag{1}$$

$U_s(r)$ is a scalar potential that is a function of r magnitude only. $U_v(r)$ is the time component of a vector potential, a function of separation only. The general tensor potential is written as

$$\sigma^{\mu\nu}U_{t\mu\nu} = (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)U_{t\mu\nu}/2i \tag{2}$$

This tensor potential is antisymmetric in its two space-time indices. With parity, total angular momentum, and its z component observed, the only allowed terms remaining of the tensor potential involve one index to be zero, corresponding to time, and the other index being a space index. The allowed combination by the above conservation laws is a radial component of the tensor potential that is a coefficient of $\gamma^0\boldsymbol{\gamma}\cdot\hat{r}$. The tensor potential component in this radial direction can be a function of separation only. The nonvanishing part becomes $\gamma^0\boldsymbol{\gamma}\cdot\hat{r}T(r)$, where the nonvanishing components are written as

$$iU_{tj} = U_{t0j} - U_{tj0} \tag{3}$$

In the electromagnetic field case, the corresponding tensor is $F^{\mu\nu}$ and the corresponding nonvanishing components would be

$$F^{0j} - F^{j0} = -2iE_j \tag{4}$$

which is the jth component of the electric field. $U_{tr}(r)$ is the radial component of a tensor potential, and is required by Hermiticity and time-reversal invariance to be pure imaginary and a function of r magnitude only. This tensor potential is analogous to a Pauli or an anomalous magnetic moment interaction in electromagnetism. The tensor potential is written as

$$U_{tr}(r) = -iT(r) \tag{5}$$

The QCD equations utilize a color vector potential, with no scalar potential, so the possible scalar potential allowed in the Dirac equation, $U_s(r)$, is assumed to be zero. The time component of a vector potential, $U_v(r)$, by itself, cannot absolutely confine a fermion, as it suffers from the Klein paradox. This potential component is therefore assumed to be zero in this work.

From the correspondence noted between equations (3) and (4), the equation for the one-body tensor potential is modeled from $\nabla\cdot E = \rho$, by asking that the divergence of the radial component of the tensor potential to be proportional to the quark color charge density, which is assumed proportional to the electromagnetic charge density, as inferred from nucleon electromagnetic form factor fits. Thus the tensor potential radial component is determined from

$$\nabla\cdot\hat{r}T = T_0\exp(-\alpha r) \tag{6}$$

The dimensionality of this divergence for linear field equations is $d = 3$. The self-interactions of the nonlinear QCD equations are thought to keep the field lines confined to nearly linear regions of space. In d dimensions, this divergence equation is solved as

$$T = T_0 r^{(1-d)}\int_0^r \exp(-\alpha r)r^{d-1}dr \tag{7}$$

Nonlinear quantum field effects may change the dimensionality. The one-body tensor potential is assumed to partially fill (Nakamura, 1993) the space around the quark charges with a characteristic fractal dimension d. That dimension is viewed as a model parameter here, along with the potential strength parameter T_0. The nonlinear interaction of the QCD theory is not solved for here. These nonlinear interactions are modeled here by the dimensionality parameter d being less than the three expected in a linear theory. With the source term being proportional to the color charge density, the potential will be a monotonic, nondecreasing function of r, only for d less than or equal to one. For d exactly one, the tensor potential approaches a constant for large separations. The tensor potential will diverge with large separations if d is less than one. For the case of d exactly one, in the limit of α goes to zero, the tensor potential goes over to the Dirac oscillator model (Moshinsky and Szczepaniak, 1989; Benitez *et al.*, 1990). Tensor potentials associated with d = 3, 2, 1, and 2/3 are shown in Fig. 1. Nonmonotonic behavior is seen for d greater than one. The desired attractive monotonic, nondecreasing behavior of T is attained if d is one or less and T_0 positive. For a dimensionality of one, the tensor potential can be thought of as existing in lines between the quarks of a system, whereas for a d of 3, the potential can be thought of as existing in a spherical volume centered about each quark. For a dimensionality of 2/3 the potential can be described as existing in chains of beads along lines between quarks.

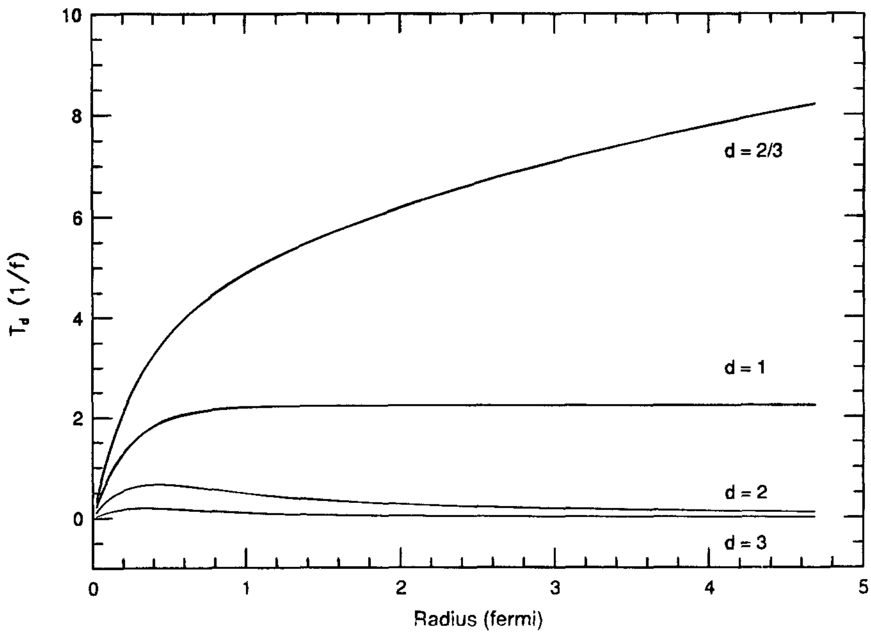

Fig. 1. Tensor potentials for various values of the dimensionality.

Expanding the quark wave function into states of good angular momentum, parity, and energy, we have

$$\psi = (1/r)\begin{pmatrix} FY_{jm}^{\omega} \\ iGY_{jm}^{-\omega} \end{pmatrix} \tag{8}$$

where F and G are the large and small component radial wave functions. Parity is given by $(-1)^{j+\omega/2}$; ω has the value plus or minus one, and is minus one for the ground state.

Defining

$$K = \omega(j + 1/2) \tag{9}$$

we find, by substituting equation (8) into the Dirac equation, the large and small component radial wave equations

$$\begin{aligned} F' + [(T/\hbar c) + K/r]F &= [(E + mc^2)/\hbar c]G \\ -G' + [(T/\hbar c) + K/r]G &= [(E - mc^2)/\hbar c]F \end{aligned} \tag{10}$$

The dipole fit to the form factor of the proton (Bosted *et al.*, 1992) suggests for the electric charge density

$$\rho = \exp(-\alpha r) \qquad \text{where} \quad \alpha = 842.6 \text{ MeV}/\hbar c \tag{11}$$

Assuming that the quark distribution in a nucleon makes the dominant contribution to the charge distribution measured, this implies that an exponentially damped quark radial wave function occurs in the nucleons. Thus F and G would have the form of being proportional to $r \exp(-ar)$, where a is $\alpha/2$. What sort of potential T will bring that about? The second-order differential equation for F is, setting $\hbar$ and c to 1,

$$-F'' + [T^2 - T' + 2KT/r + K(K+1)/r^2]F = (E^2 - m^2)F \tag{12}$$

The case of $K = -1$, the ground state, is considered now. If T goes to zero for large r, then E^2 is less than m^2 in a bound-state wave function. But the tensor potential going to zero implies a potential in contradiction to a monotonic, nondecreasing attractive potential. If T goes to a constant, then E^2 is greater than m^2, which also allows a bound-state solution to equation (12) if the constant value is large enough. This situation is compatible with a dimensionality-one solution of equation (7). This dimensionality will permit a self-consistent solution of equations (7) and (10) to be obtained. To absolutely confine quarks in this model, the dimensionality d must be less than one. Such values, however, do not result in exponentially damped radial equations for the quark wave functions. Those cases will be considered separately. For $K = -1$, the effective potentials of (12) for various values of dimensionality

d can be seen in Fig. 2. The dimensionality has to be 1 or less for these effective potentials to be nondecreasing, attractive potentials. The effective potential appearing in equation (12) for a d of 2/3 is very similar to a potential previously used by Fabre del la Ripelle (1988) in fitting meson spectra with a Schrödinger equation approach. For dimensionalities less than one, this tensor potential will confine quarks in all states where K is a positive or negative integer.

3. THE SELF-CONSISTENT SOLUTION

If d is exactly one, a self-consistent solution can be found where the mass density ρ is taken as the exponentially damped quark density, which is used as the radial dependence of the source for the radial tensor potential, which is then used to determine the quark wave functions self-consistently. With $T_0 = 0.5205\alpha$, the $K = -1$, ground-state solution of equation (12) decays exponentially. Figure 3 shows the large and small component radial wave functions. Both components closely resemble the analytic form of r times an exponential. By adjusting the quark energy and mass, the relative normalization of F and G has been chosen to reproduce the axial vector coupling constant. The numerical value found for $E^2 - m^2$ is $0.38108(1/f^2)$.

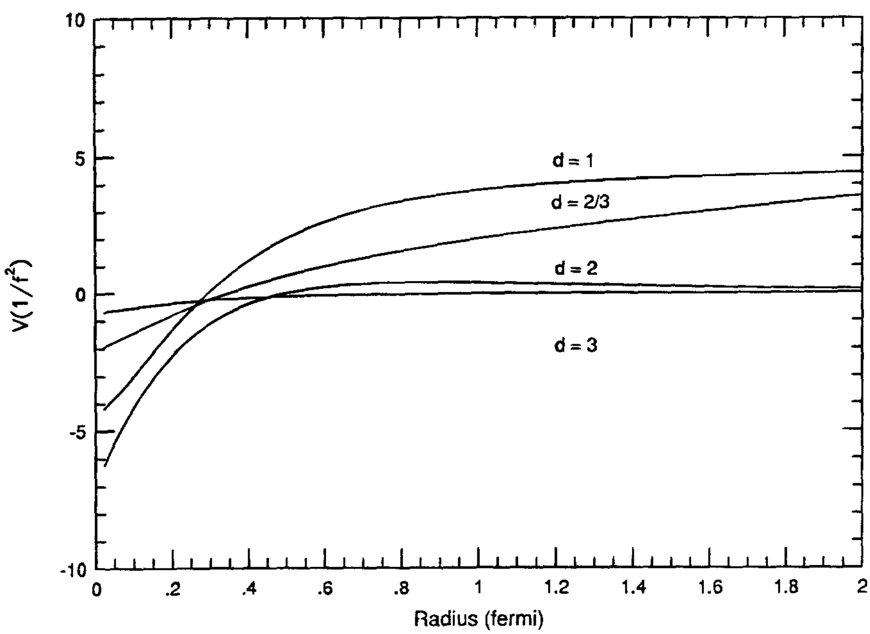

Fig. 2. Effective radial positive energy potentials for various values of the dimensionality.

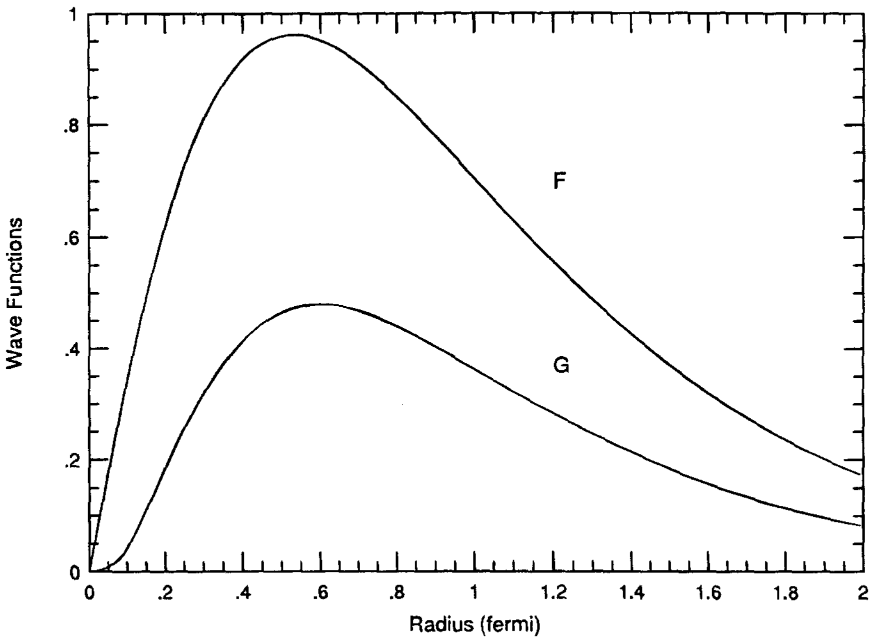

Fig. 3. Large and small component radial wave functions with mass chosen to fit the axial vector coupling constant.

The calculated electric charge density is smaller than the dipole fit for distances less than $^1/_2$ fermi. The calculated charge density smoothly matches the dipole fit for larger distances. The ratio is a constant for distances larger than 1 fermi. This tensor potential has resulted in a calculated charge density with the same exponential shape as the charge distribution used to generate the tensor potential. This is then a self-consistent calculation.

4. SUMMARY

A one-body tensor potential is used to bind quarks in a Dirac equation approach. The divergence of the tensor potential is taken proportional to the quark color charge density. A fractal dimension to the divergence is invoked to produce attractive monotonic, nondecreasing one-body potentials. A dimension of one or less will satisfy this criterion. A dimension of exactly one permits a self-consistent solution using an exponential quark density inferred from a dipole fit to the electromagnetic form factors. A dimension of two-thirds results in an effective potential similar to one used previously to fit meson and baryon excited-state spectra.

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